2016 理工 問4.

[1]

$G_X[S] = \exp\{\lambda(S-1)\}$　$S = e^t$ とすればok

$G'_X[S] = e^{-\lambda}\cdot\lambda e^{\lambda S}\Big|_{S=1}$　$\lambda = E[X]$

$G''_X[S] = e^{-\lambda}\cdot\lambda^2 e^{\lambda S}\Big|_{S=1}$　$\lambda^2 = E[X(X-1)]$

$V[X] = \lambda^2 + \lambda - \lambda^2 = \lambda$

[2]

$P(x+1) \geq P(x)$

$\Leftrightarrow \dfrac{\lambda^{x+1}}{(x+1)!}e^{-\lambda} \geq \dfrac{\lambda^x}{x!}e^{-\lambda} \Leftrightarrow \dfrac{\lambda}{x+1} \geq 1 \Leftrightarrow \lambda - 1 \geq x$　λが整数なら等号成立で $P(x+1) = P(x)$.

$x = [\lambda - 1]$，$\lambda \in \mathbb{Z}$ なら $x = \lambda - 1, \lambda$

[3]

$L(\lambda) = \prod \dfrac{1}{x_i!}\cdot\lambda^{x_i}\cdot e^{-\lambda} = \square\cdot\lambda^{\Sigma x_i}\cdot e^{-n\lambda}$

$l(\lambda) = \log\square + \Sigma x_i \log\lambda - n\lambda$

$l'(\lambda) = \dfrac{\Sigma x_i}{\lambda} - n = 0.$　$\hat{\lambda} = \dfrac{1}{n}\Sigma x_i$

表より,

$\Sigma x_i = 0\cdot 8 + 1\cdot 15 + 2\cdot 12 + 3\cdot 10 + 4\cdot 3 + 5\cdot 2 = 15 + 24 + 30 + 12 + 10 = 91.$

$n = 50$ より

$\hat{\lambda} = \dfrac{91}{50} = 1.82.$

[4]

$G_Y[S] = \dfrac{e^{-\lambda}}{1-e^{-\lambda}}\sum_{1}^{\infty}\dfrac{(S\lambda)^y}{y!} = \dfrac{e^{-\lambda}}{1-e^{-\lambda}}(e^{S\lambda} - 1)$　$M_Y[t] = \dfrac{e^{-\lambda}}{1-e^{-\lambda}}\exp[\lambda e^t - 1]$

$G'_Y[1] = \dfrac{e^{-\lambda}}{1-e^{-\lambda}}\lambda e^{S\lambda}\Big|_{S=1}$ $\dfrac{\lambda}{1-e^{-\lambda}}$

2016 理工 問3

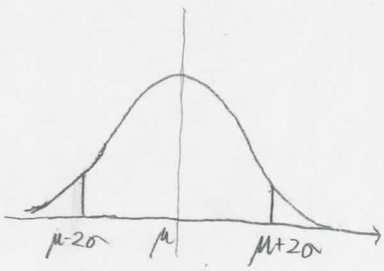
$$C_p = \frac{S_U - S_L}{6\sigma}, \quad \hat{C}_p = \frac{S_U - S_L}{6S}$$

[1] 正規分布に従っており，正規分布はμを中心に左右対称だから。

[2]

$$\left.\begin{array}{l} \dfrac{S_U - S_L}{6\sigma} = \dfrac{2}{3} \Leftrightarrow S_U - S_L = 4\sigma \\ \dfrac{S_U + S_L}{2} = \mu \Leftrightarrow S_U = 2\mu - S_L \end{array}\right\}$$

$$2\mu - S_L = S_L + 4\sigma \Leftrightarrow \left.\begin{array}{l} \mu - 2\sigma = S_L \\ \mu + 2\sigma = S_U \end{array}\right\}$$



$$0.228 \times 2 = 0.456$$

[3]

$$\hat{C}_p \sqrt{\frac{\chi^2_{0.975}(n-1)}{n-1}} < C_p < \hat{C}_p \sqrt{\frac{\chi^2_{0.025}(n-1)}{n-1}}$$

$$\Leftrightarrow \frac{S_U - S_L}{6S}\sqrt{} < \frac{S_U - S_L}{6\sigma} < \frac{S_U - S_L}{6S}\sqrt{}$$

$$\sqrt{} < \frac{S}{\sigma} < \sqrt{}$$

$$S^2 = \frac{1}{n-1}\Sigma(X - \bar{X})^2$$

$$\frac{1}{n-1}\chi^2_{0.975}(n-1) < \frac{S^2}{\sigma^2} < \frac{1}{n-1}\chi^2_{0.025}(n-1)$$

$$\chi^2_{0.975}(n-1) < \frac{\Sigma(x_i - \bar{x})^2}{\sigma^2} < \chi^2_{0.025}(n-1)$$

$$< \quad Y_{n-1} \quad < \qquad\qquad Y_{n-1} \sim \chi^2(n-1)$$

[4] $S_U = 12.6 \quad S_L = 12.0 \quad n = 20 \quad \sigma = 0.05$

$$\hat{C}_p' = \frac{12.6 - 12.0}{6 \cdot 0.05} = \frac{0.6}{0.3} = 2,$$

$$2 \cdot \sqrt{\frac{\chi^2_{0.975}(19)}{19}} < C_p < 2\sqrt{\frac{\chi^2_{0.025}(19)}{19}}$$

$(1.33 <)$ $1.37 < C_p < 2.63$. 優良な工程.

[5]

$$E[\hat{C}_p] = E\left[\frac{S_U - S_L}{6S}\right] = E\left[\frac{S_U - S_L}{6} \cdot \frac{\sigma}{\sigma} \cdot \frac{1}{S} \cdot \frac{n-1}{n-1}\right] = \frac{\sqrt{n-1}}{6\sigma}(S_U - S_L) E\left[\left\{\frac{(n-1)S^2}{\sigma^2}\right\}^{-1}\right] = C_p\sqrt{n-1}\, E\left[\frac{1}{\sqrt{Y_{n-1}}}\right]$$

$$E\left[\frac{1}{\sqrt{Y}}\right] = \frac{1}{2\Gamma\left(\frac{n-1}{2}\right)}\int_0^\infty \left(\frac{1}{2}\right)^{\frac{n-1}{2}-1} y^{\frac{n-2}{2}-1} \cdot e^{-\frac{y}{2}} dy = \left(\frac{1}{2}\right)^{\frac{n-3}{2}} \Gamma\left(\frac{n-1}{2}\right)^{-1} \int_0^\infty (2z)^{\frac{n-2}{2}-1} e^{-z} \cdot 2dz = 2^{-\frac{n-3}{2}} \cdot 2^{\frac{n-2}{2}} \frac{\Gamma\left(\frac{n-2}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}$$

$$E[\hat{C}_p] = C_p\sqrt{n-1} \cdot \frac{1}{\sqrt{2}} \frac{\Gamma\left(\frac{n-2}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}$$